

Custom Nonlinear Elements for Chaotic Circuits

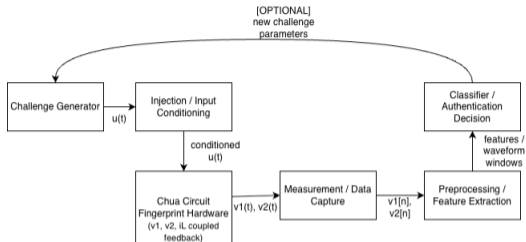
MING: A Novel Nonlinear Chua-Style Diode

Divit Rawal David Kim

PHYSICS 111A: Instrumentation Lab
University of California, Berkeley

May 2026

Block Diagram



Block diagram of the challenge-response fingerprinting pipeline.

Optional feedback

If confidence is low, the classifier can request new challenge parameters and repeat the measurement.

Core idea

A controlled challenge signal $u(t)$ is injected into a Chua circuit. Because component tolerances and nonlinear feedback affect the chaotic dynamics, each hardware instance produces a slightly different response.

- The challenge generator applies a known input $u(t)$. The injection network safely couples the input into the circuit.
- The Chua circuit produces chaotic outputs $v_1(t)$ and $v_2(t)$ through internal coupled feedback between v_1 , v_2 , and i_L .
- The measured signals are digitized as $v_1[n]$ and $v_2[n]$. Features or waveform windows are classified to identify or authenticate the circuit.

Chua Diode

- Chua's circuit is a third-order nonlinear dynamical system.
- The capacitors and inductor provide the state variables; the Chua diode supplies the nonlinear active element.
- The key circuit feature is **negative differential resistance**: over part of the I - V curve, increasing voltage decreases current.
- This nonlinear feedback enables non-periodic chaotic motion instead of simple decay or sinusoidal oscillation.

Ideal Chua diode characteristic

$$i(v_R) = m_1 v + \frac{1}{2}(m_0 - m_1)(|v + B_p| - |v - B_p|)$$

$$m_0 < 0, \quad m_1 < 0, \quad |m_0| > |m_1|$$

Our goal

A tunable two-terminal element with a measured negative-slope I - V region and controllable breakpoints.

Kennedy's Two-Op-Amp Chua Diode

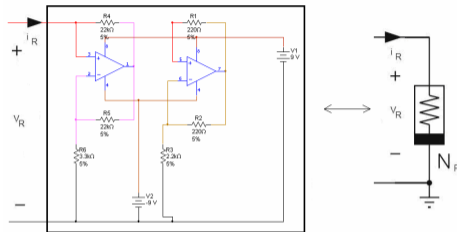


Figure 5. Chua diode realized using M.P. Kennedy's two op-amp realization

Kennedy Chua diode schematic/model.

How the model works

- Uses op-amp stages and resistor networks to synthesize a piecewise-linear negative-resistance element.
- Saturation/clipping in the op-amp paths creates the breakpoints $\pm B_p$.
- The effective slopes m_0 and m_1 are set by resistor ratios.

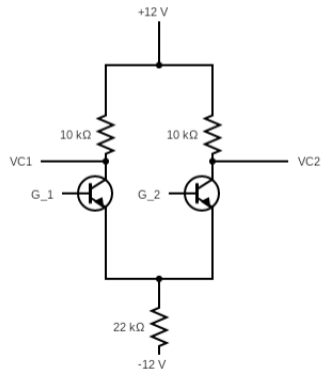
Why we built it first

Kennedy's Chua diode gives us a known nonlinear element for validating the $I-V$ measurement workflow before developing our custom MING diode.

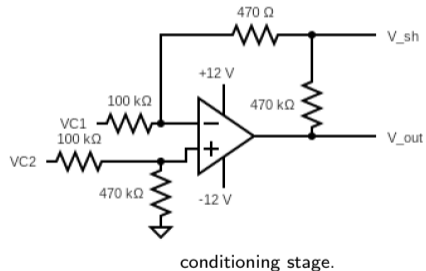
The MING Diode

Matched-pair Inverting Nonlinear Generator

The MING diode is our custom nonlinear two-terminal element. We use matched JFET behavior, op-amp signal conditioning, and bias networks to create a tunable nonlinear port.



Matched JFET stage.



Op-amp

conditioning stage.

On the Design of the MING Diode

- The JFET pair creates a nonlinear current response because drain current depends nonlinearly on gate-source voltage.
- The op-amp stage inverts and scales the signal, turning the device-level nonlinearity into a usable two-terminal port characteristic.
- Bias and limiting networks shift the operating point and breakpoint placement.

JFET	Current (mA)
1	1.74
2	1.76
3	1.50
4	1.66
5	1.54

Chosen pair

JFET 1 and JFET 2 were selected because their measured currents were closest.

Generating Nonlinearity in the MING Diode

- Each JFET has a nonlinear drain current response controlled by its gate-source voltage:

$$I_D \approx I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2.$$

- Driving the two gates with opposite or shifted control voltages produces a differential nonlinear response.
- The op-amp stage senses this imbalance, inverts it, and scales it into a voltage signal.
- That conditioned voltage is injected back through resistors into the external port, so the port behaves like a nonlinear resistor.

How this imitates a Chua diode

A Chua diode need not be a literal diode. It only needs an I - V curve with a controllable negative-slope region:

$$\frac{dI}{dV} < 0$$

over the oscillator's operating range.

In a nutshell

JFET nonlinearity \rightarrow op-amp inversion/scaling \rightarrow injected port current \rightarrow effective negative differential resistance.

MING Diode I - V Curve

Device-level nonlinearity

Gate-source voltage modulates the conducting channel. As V_{GS} becomes more negative, the channel pinches off and the drain current decreases approximately quadratically:

$$I_D \approx I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2.$$

- Small changes in port voltage shift the JFET gate voltages.
- That changes the channel conductance, so the current response is not proportional to voltage.
- This gives a curved transfer relation instead of a linear resistor law $I = V/R$.

Negative differential resistance

The circuit is biased so that increasing the port voltage causes the active JFET/op-amp path to inject current in the opposite direction:

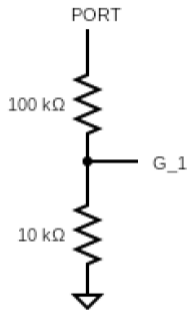
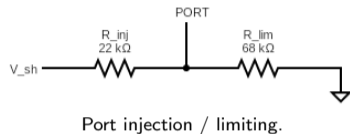
$$\frac{dI_{\text{port}}}{dV_{\text{port}}} < 0.$$

In that region, the active feedback contribution is stronger than the passive positive conductance, so the net slope becomes negative.

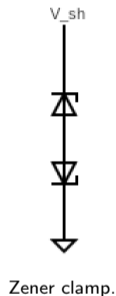
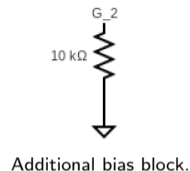
Op-amp

The op-amp inverts and scales the JFET imbalance.

MING Port, Limiting, and Bias Networks

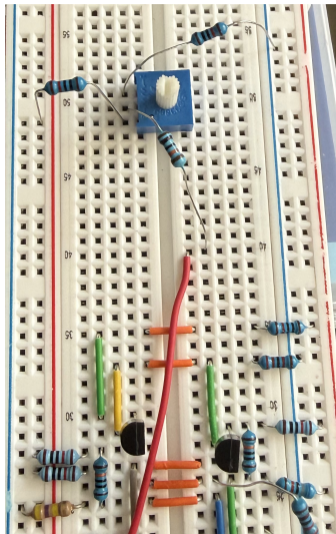


Gate 1 bias network.



- Separate subcircuits let us tune the nonlinear port rather than accepting the raw JFET response.
- The bias networks move the operating window; the limiting/injection network controls how strongly the nonlinear response enters the port.

Tuning the MING Diode

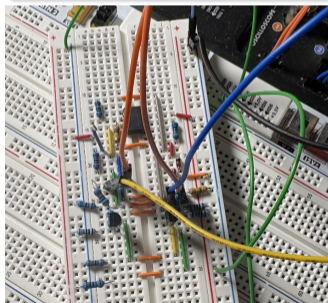


Bias

tuning voltage divider.

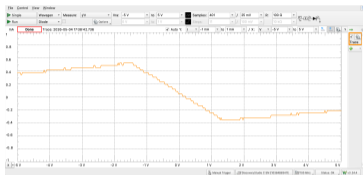
Tuning DC V offset

We used a potentiometer to tune the DC voltage offset so that we could ensure the diode's I-V curve passed through the origin (necessary for the Chua diode to function correctly).

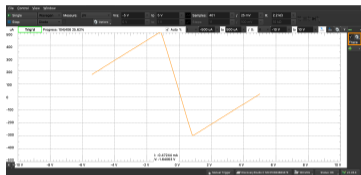


MING diode and I-V curve measurement setup.

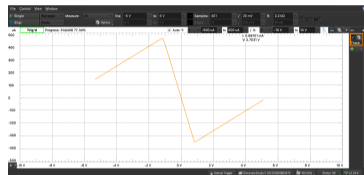
Measured $I-V$ Curves: Kennedy Chua's vs. MING



Kennedy Chua diode.



Untuned MING $I-V$.



Tuned MING $I-V$.

- Both diodes satisfy the expected nonlinear signature.
- Tuning potentiometer allows us to center $I-V$ curve.
- The tuned MING curve shows a strong, centered negative-slope region, making it a plausible Chua-style nonlinear element.

$$I_{port} = \frac{V_{sense}}{R_{sense}}, \quad V_{port} = V_{drive} - V_{sense}$$

Extracted Tuned MING I - V Parameters

Measured points used for piecewise-linear estimate

$(-5.30 \text{ V}, 140 \mu\text{A}), (-1.15 \text{ V}, 460 \mu\text{A}), (0.84 \text{ V}, -350 \mu\text{A}), (5.10 \text{ V}, -20 \mu\text{A})$

Extracted parameters

$$\begin{aligned} B_- &\approx -1.15 \text{ V}, & B_+ &\approx +0.84 \text{ V} \\ m_0 &\approx -0.41 \text{ mS}, & r_d = \frac{1}{m_0} &\approx -2.45 \text{ k}\Omega \\ m_1 &\approx +0.077 \text{ mS} \end{aligned}$$

- The central segment has negative differential resistance.
- The breakpoint asymmetry is due to bias offset and nonideal JFET/op-amp behavior.

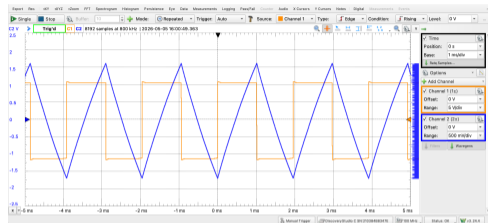
Schmitt-Trigger RC Oscillator

Motivation

Building a reliable mH-scale inductor proved difficult (toroid underperformed, coiled inductors hard to solder, gyrator bandwidth-limited).

- A Schmitt trigger with hysteresis thresholds V_{TH}^+ and V_{TH}^- drives an RC network.
- The capacitor charges and discharges between the two thresholds, producing a square-wave output and a triangular capacitor voltage.
- Oscillation period:

$$T \approx 2RC \ln \left(\frac{V_{TH}^+ - V_{sat}^-}{V_{TH}^- - V_{sat}^-} \right)$$



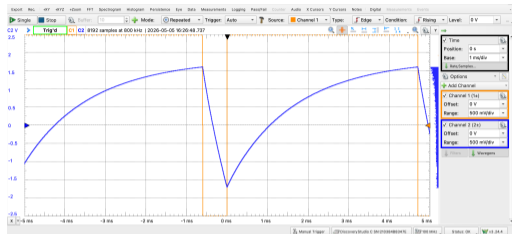
Baseline Schmitt-trigger RC relaxation oscillator: periodic square/triangle output.

State variables (baseline)

$$x_1 = v_C(t), \quad x_2 = \text{Schmitt state} \in \{H, L\}$$

Only 2 variables \Rightarrow no chaos possible.

MING-Modified Schmitt RC



Schmitt RC oscillator with the MING diode inserted into the timing path.

State variables (modified)

$$x_1 = v_C, \quad x_2 = \text{Schmitt state}, \quad x_3 = \text{MING internal state}$$

The MING diode's nonlinear feedback adds the third independent state variable required for chaos.

The MING effect

The MING diode replaces the constant resistor with a nonlinear, voltage-dependent conductance in the RC charging path - capacitor charge/discharge is no longer on a simple exponential.

- The triangular $v_C(t)$ becomes distorted, reflecting the MING $I-V$ piecewise structure.
- Negative differential resistance accelerates charge/discharge in part of the cycle, raising the effective oscillation frequency.
- Variation in JFET match, bias, and breakpoints produces unique per-circuit waveform fingerprints. Not quite chaotic but good indicator for fingerprinting.

Chua's Circuit

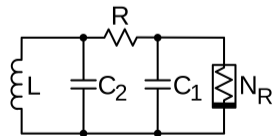
- Chua's circuit combines two capacitors, an inductor, a linear resistor, and a nonlinear resistor.
- The energy-storage elements make the system third order, which is the minimum order for autonomous continuous-time chaos.
- The linear resistor couples the capacitor voltages; the inductor current supplies the third state variable.
- The nonlinear diode determines whether trajectories settle, oscillate periodically, or enter a chaotic regime.

State variables

$$x = v_{C1}, \quad y = v_{C2}, \quad z = i_L$$

Practical sensitivity

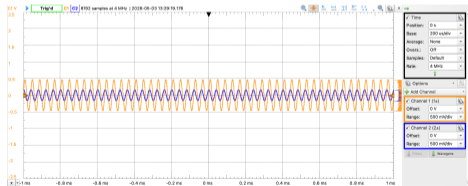
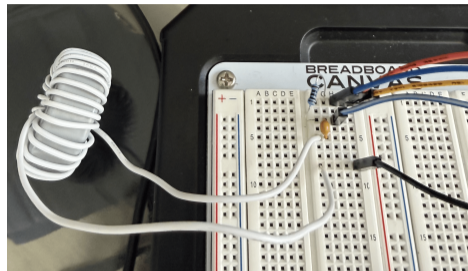
Breadboard chaos depended strongly on inductor value, parasitic resistance, op-amp limits, and exact nonlinear breakpoint tuning.



Chua's circuit.

Inductor Candidates: Toroid, Coiled Inductor

- To have an inductor in the 10s of mH range, the first candidate was the toroid with 26 loops of wire wrapped.
- The inductance was measured with an RLC circuit shown on the right, yielding around $400\mu\text{H}$.
- The image below shows an attempt to solder coiled 1.4mH inductors in parallel, but was scrapped due to difficulty.

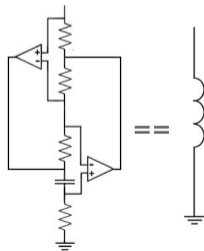
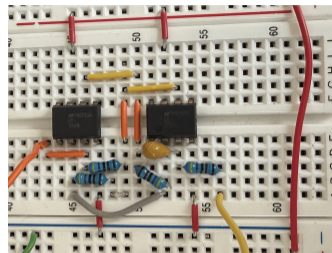


Inductor Candidates: Antoniou Gyrator

Antoniou gyrator effective inductance

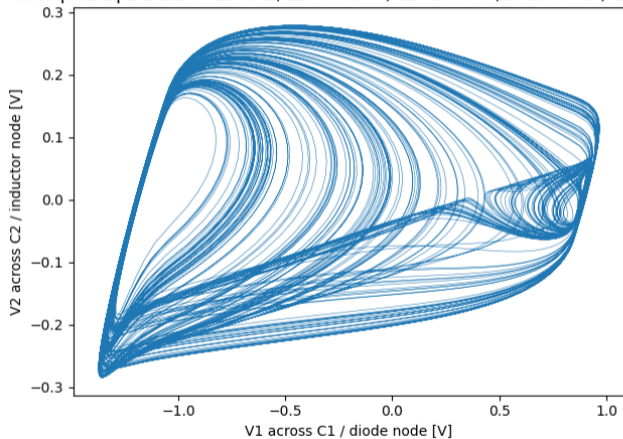
$$L_{eq} = \frac{R_1 R_3 R_5 C}{R_2}$$

- The Antoniou gyrator uses two op-amps, four resistors, and one capacitor to synthesize an effective inductance.
- The op-amp network adjusts voltages on the capacitor nodes so that $i_C \propto \frac{di_{in}}{dt}$ and the resistor ladder converts current to voltage $v_{in} \propto i_C$.
- The synthesized inductor only behaves correctly over a limited frequency range, $f_{usable} \ll \frac{GBW}{\text{Noise Gain}}$. Usually operating frequency of 100 times lower than the op-amp closed loop bandwidth preferred.



Chaotic Behavior Results (Simulated)

Chua phase portrait: $R=2500\ \Omega$, $C1=3.00\ \text{nF}$, $C2=33.0\ \text{nF}$, $L=16.0\ \text{mH}$, $rL=5.0\ \Omega$



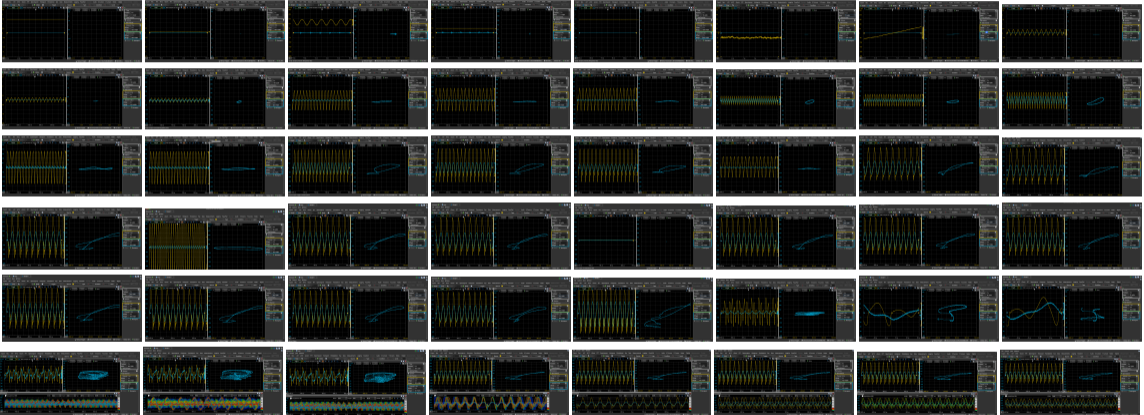
Chua circuit with MING diode simulated phase diagram.

Chaotic Behavior Results



Nearly chaotic results; spontaneously switching between many stable modes.

Chaotic Behavior Results :(



References



M. P. Kennedy, "Three Steps to Chaos—Part II: A Chua's Circuit Primer," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 40, no. 10, pp. 657–674, 1993.



L. O. Chua, C. W. Wu, A. Huang, and G. Q. Zhong, "A Universal Circuit for Studying and Generating Chaos—I: Routes to Chaos," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 40, no. 10, pp. 732–744, 1993.



E. Nazarenko, N. A. Anagnostopoulos, S. G. Stavrinos, N. Mexis, F. Frank, T. Arul, and S. Katzenbeisser, "Real-World Chaos-Based Cryptography Using Synchronised Chua Chaotic Circuits," arXiv:2210.11299, 2022.



J. Zhang, *Hardware and Software Fingerprinting of Mobile Devices*, Ph.D. dissertation, University of Cambridge, 2021.